Efficient nondestructive equality checking for trees and graphs

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Overview

Background

Revised[5] Report on the Algorithmic Language Scheme (R5RS):

- "*Equal?* may fail to terminate if its arguments are circular data structures"

Background

Scheme Request for Implementation 85 (SRFI-85):

- Defines *equiv?*
- Terminates on all input
- Reference implementation

Background

Revised^6 Report on the Algorithmic Language Scheme (R6RS):

- "The *equal?* predicate returns #t if and only if the (possibly infinite) unfoldings of its arguments into regular trees are equal as ordered trees."

Requirements

- Correct
- Unknown inputs
- Not slow

R5RS (Tree) Equality

R5RS (Tree) Equality

```
(define (equal? x y)
  (cond
    [(eq? x y) #t]
    [(pair? x) (and (pair? y)
                    (equal? (car x) (car y))
                    (equal? (cdr x) (cdr y)))]
    [(vector? x) (and (vector? y)
                      ...)]
    ...))
```

R5RS (Tree) Equality

- On non-sharing:
  - Fast

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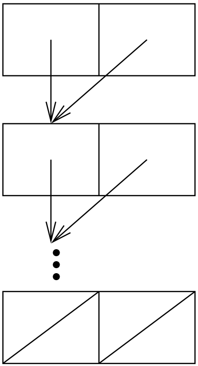
- On cycles:
  - Loops forever

R5RS (Tree) Equality

- On non-sharing:
  - Fast
- On cycles:
  - Loops forever
- On acyclic sharing:
  - Exponential

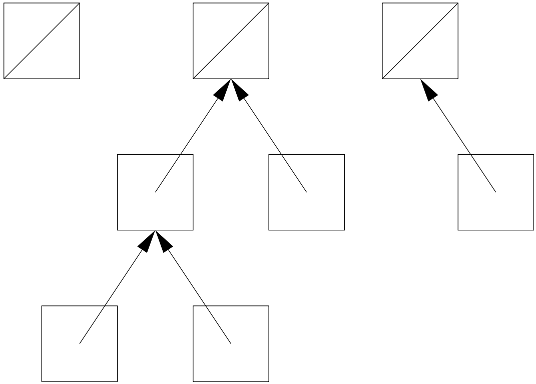


Union-Find (Graph) Equality

Union-Find: Algorithm

- Equivalence classes
- Each call
  - Trivial
  - Same equivalence class
  - Different equivalence classes

Union-Find



Union-Find: Analysis

- Union-find
  - Inverse Ackermann (almost constant)
- Equality
  - Almost linear
- DFA equivalence algorithms

Union-Find: Implementation

- Pointer per object
- Store in object header
  - Not pure/functional
  - Overhead to every object
  - Threads

Union-Find: Implementation

- Pointer per object
- Store in object header
  - Not pure/functional
  - Overhead to every object
  - Threads
- Use "eq" hash tables
  - Map object to pointer
  - GC Interaction

Union-Find: Performance

- 25x slower than tree-equality
    - Hash table
    - Union-find algorithm

Precheck

Precheck

Algorithm

- Tree-equality for N steps
- After N steps, restart and run union-find

- Basic idea in SRFI-85

Setting Precheck Bound

- Bound too big
- Bound too small
- No perfect bound

Interleaved Equality

Interleaved

- Want
  - Fast tree-equal on trees
  - Required union-find-equal on cycles/DAGS
- Idea
  - Co-routines

Interleaved

- Tree-equality for k_0 steps
- Union-find for k_b steps
    - But continue if successful

Interleaved

- Tree-equality for k_0 steps
- Union-find for k_b steps
  - But continue if successful
- Synergy
  - Tree parts get tree-equality
  - Cycle and graph parts get union-find

Analytic Performance

- Within constant of union-find
- Start with union-find
  - k_0/k_b
- Start with tree mode
  - k_0

Where do we start?

- Union-find
  - Pro: k_0/k_b, large cycles
  - Con: small trees
- Tree
  - Pro: small trees
  - Con: k_0, large cycles

Where do we start?

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  - Pro: small trees
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Neither solution acceptable

Interleaved with Precheck

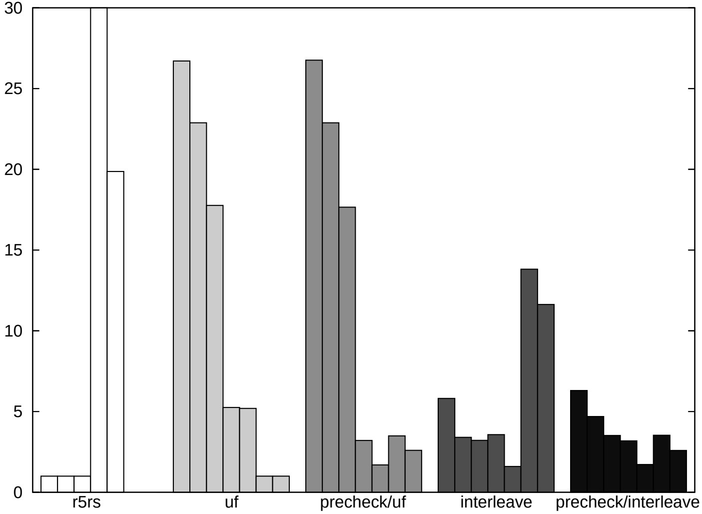
Interleaved with Precheck

- Interleave *and* precheck
  - Small trees
  - Large cycles

Benchmarks

- Lists
- Rev. lists
- Rand. DAGS
- Balanced binary tree
- Degenerate DAGS
- Random graph
- Union-exercising graph

Benchmark


30
25
20
15
10
5
0
r5rs
uf
precheck/uf
interleave
precheck/interleave

Conclusions

- Simple idea, subtle performance implications
- Mixing best of all worlds
    - Tree
    - Union-find
    - Interleaving
    - Precheck

Conclusions

- Interleaving with precheck
  - Never the best
  - Always almost the best
- Particular applications vs. General purpose library

Questions?